Appendix 2—Complete derivation of equation 7

Song and Graf (1995) proposed an equation to estimate the friction velocity (u*) of a gradually varying, unsteady flow (flow depth varies along the channel length and with time [Chow, 1959, p. 5-6]) as:

$$u^{*2} = gy\mathrm{S}_0 + \left\{-gy\frac{\Delta y}{\Delta x}(1-\mathrm{Fr}^2)\right\} + \left(u\frac{\Delta y}{\Delta t} - y\frac{\Delta u}{\Delta t}\right) \quad \text{(I).}$$

Where *t* is time in seconds and $\Delta y/\Delta t$ and $\Delta u/\Delta t$ designates rates of change in *y* and *u*, respectively, with respect to *t*. In this study, the flow was gradually varied and steady, where flow depth varied along the channel length, but not with time; therefore,

$$\Delta y/\Delta t = \Delta u/\Delta t = 0 \quad \text{(II).}$$

Further, as in Song and Graf (1995):

$$u^* = \sqrt{\tau_0/\rho_\mathrm{w}} \quad \text{(III),}$$

and *y* can be substituted with R (Song and Chiew, 2001).

Therefore, equation 20 becomes:

$$\frac{\tau_0}{\rho_\mathrm{w}} = g\mathrm{RS}_0 + \left\{-g\mathrm{R}\frac{\Delta y}{\Delta x}(1-\mathrm{Fr}^2)\right\} \quad \text{(IV).}$$

Rearranging equation 23 in terms of τ_0, and substituting τ_0 with τ_c at eggshell deposition yields:

$$\tau_\mathrm{c} = \left[\mathrm{S}_0 - \frac{\Delta y}{\Delta x}(1-\mathrm{Fr}^2)\right]\rho_\mathrm{w} g\mathrm{R} \quad \text{(V),}$$

where Fr was Froude number and R was hydraulic radius of the flow at each eggshell deposition. Fr is defined as (Chow, 1959, p.45):

$\mathrm{Fr} = \frac{u}{\sqrt{g \cdot y}}$ (VI).

In equation VI, u was calculated as Q divided by cross-sectional area of the flow (A):

$u = \frac{\mathrm{Q}}{\mathrm{A}} = \frac{\mathrm{Q}}{0.46y}$ (VII),

and equation VI became:

$Fr = \frac{\mathrm{Q}}{0.46y\sqrt{g \cdot y}}$ (VIII).

For the rectangular flume in this study (flume width = 46 cm), R was calculated as (Chow, 1959, table 2-1):

$\mathrm{R} = \frac{0.46y}{0.46+2y}$ (IX).

At a given x, $\Delta y/\Delta x$ was approximated by using the centered finite-divided-difference fomula as (Chapra and Canale, 1988: fig. 23.3):

$\frac{\Delta y}{\Delta x} = \frac{y_{x+1}-y_{x-1}}{2x}$ (X),

where y_{x+1} and y_{x-1} were y at (x + 30 cm) and (x – 30 cm), respectively. Therefore, by equations V, VIII, IX, and X, τ_c was function of two variables, x and y, as:

$\tau_c = \left[\mathrm{S}_0 - \frac{y_{x+1}-y_{x-1}}{\mathbf{2}x}\left(1 - \frac{\mathrm{Q}^2}{(0.46y)^2 gy}\right)\right] \rho_w g \frac{0.46y}{0.46+2y}$ (7).